Date 21/2/25 Sem:- IV MJC PHY 06 Unit:-1

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Poynting's Theorem:-

Since total Energy stored in electro-magnetic field is

$$U = U_m + U_E$$

$$= \frac{1}{2}\int_{all\ space} \left(\frac{1}{\mu_0} B^2 + \epsilon_0 E^2\right) dV \quad \text{——— (1)}$$

Let us suppose that some distribution of charges & currents in small time dt a charge will move $\underline{v}dt$ and according to Lorentz force law,

The work done on the charge will be

$$dU = \underline{F}\cdot \underline{dl} = q(E + \underline{v}\times\underline{B})\cdot \underline{v}dt = qE\cdot v dt$$

where as usual the magnetic forces do no work

Let $q = \rho dV$ & $\rho\underline{v} = \underline{J}$

Then dividing through by dt and integrating over a volume V containing the charges.

Then power delivered to the system

$$\boxed{\frac{dU}{dt} = \int_V E\cdot \underline{J}\, dV} \quad \text{——— (2)}$$

Thus $\underline{E}\cdot\underline{J}$ is the power delivered per unit volume.

$$\underline{E}\cdot\underline{J} = \frac{1}{\mu_0}\underline{E}\cdot(\underline{\nabla}\times\underline{B}) - \epsilon_0\underline{E}\cdot\frac{\partial E}{\partial t}$$

$$\underline{E}\cdot(\underline{\nabla}\times\underline{B}) = \underline{B}\cdot(\underline{\nabla}\times\underline{E}) - \underline{\nabla}\cdot(\underline{E}\times\underline{B})$$

$$= -\underline{B}\cdot\frac{\partial B}{\partial t} - \nabla\cdot(\underline{E}\times\underline{B})$$

$$\Rightarrow \quad \underline{B}\cdot\frac{\partial B}{dt} = \frac{1}{2}\frac{\partial B^2}{\partial t} \qquad \underline{E}\cdot\frac{\partial E}{\partial t} = \frac{1}{2}\frac{\partial E^2}{\partial t}$$

yields

$$\underline{E}\cdot\underline{J} = -\frac{1}{2}\frac{\partial}{\partial t}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \frac{1}{\mu_0}\underline{\nabla}\cdot(\underline{E}\times\underline{B})$$

Finally, integrate over the volume V containing the currents and charges & using divergence theorem of second term to obtain from equ^n 2

$$\boxed{\frac{dU}{dt} = -\frac{\partial}{\partial t}\int_V \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)dv - \frac{1}{\mu_0}\oint_S (\underline{E}\times\underline{B})\cdot}$$

3

This is called poynting's theorem i.e. equ^n 3

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